Syllabus for 6.7720/18.619/15.070: Discrete Probability and Stochastic Processes

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Spring 2025

Lectures: Mondays and Wednesdays 2:30–4:00pm in E25–111

MIT's Course Description Provides an introduction to tools used for probabilistic reasoning in the context of discrete systems and processes. Tools such as the probabilistic method, first and second moment method, martingales, concentration and correlation inequalities, theory of random graphs, weak convergence, random walks and Brownian motion, branching processes, Markov chains, Markov random fields, correlation decay method, isoperimetry, coupling, influences and other basic tools of modern research in probability will be presented. Algorithmic aspects and connections to statistics and machine learning will be emphasized.

1 Logistics

Office Hours

- Kuikui Liu: Thursdays 2:00pm – 3:00pm in 32-D632
- Ezra Erives: Tuesdays 2:00pm – 3:00pm in 24-323
- Matthew Hong: Wednesdays 11:00am – 12:00pm in 24-323
- Anka Hu: Mondays 11:00am – 12:00pm in 24-323
- Wei Zhang: Fridays 1:00pm – 2:00pm in 24-323

Linkage

- Staff Email (please send questions/concerns here first): 67720-staff-sp25@mit.edu
- Discussion Board: https://piazza.com/mit/spring2025/6772015070
- Canvas: https://canvas.mit.edu/courses/31861
- PSET Partners: https://psetpartners.mit.edu

Additional Resources The lectures will be recorded and posted, and detailed lecture notes will be provided. The following additional references may be helpful:

- "Probability in High Dimension" by Ramon van Handel
- "High-Dimensional Probability" by Roman Vershynin
- "Concentration Inequalities: A Nonasymptotic Theory of Independence" by Gábor Lugosi, Pascal Massart, and Stéphane Boucheron
- "The Probabilistic Method" by Noga Alon and Joel Spencer

- "Probability on Graphs: Random Processes on Graphs and Lattices" by Geoffrey Grimmett
- "Markov Chains and Mixing Times" by David A. Levin, Yuval Peres, and Elizabeth L. Wilmer
- "Random Graphs" by Béla Bollobás

Deliverables

- Six Problem Sets: 40% (see below for more details)
- Take-home Midterm Exam: 35% (see below for more details)
- Final Project: 25% (see below for more details)

2 Tentative Lecture Schedule

Note that the selection of topics below is only an approximation. They are subject to change.

The Probabilistic, First Moment, and Second Moment Methods (5 Lectures)

1. Course intro, Erdös–Rényi random graph, connectivity threshold
2. Galton–Watson Branching processes, percolation on the infinite tree, Paley–Zygmund
3. Statistical inference for the broadcast process
4. Lovász Local Lemma, Boolean satisfiability
5. Shannon's Noisy Coding Theorem, Weak Law of Large Numbers

Concentration of Measure Part I (3 Lectures)

1. Chernoff–Hoeffding concentration, Central Limit Theorem, Berry–Esseen Theorem, anticoncentration
2. Sub-Gaussian random variables, maxima of stochastic processes, norm of a random matrix
3. Sub-exponential random variables, Bernstein's Inequality, Poisson limit of sparse Binomials

Martingales (2 Lectures)

1. Doob martingales, Azuma–Hoeffding, McDiarmid, stochastic Euclidean TSP
2. Stopping times, optional stopping theorem, gambler's ruin, hitting times for biased random walks

Concentration and Random Graphs (4 Lectures)

1. Independence number and chromatic number of random graphs
2. Janson's Inequality, positive correlations, lower tail for triangle counts in Erdös–Rényi
3. Giant component in Erdös–Rényi
4. Contiguity, random regular graphs, small subgraph conditioning

Markov Chains, Probabilistic Graphical Models, and Connections with Previous Topics (7 Lectures)

1. Introduction to Markov chains, random walks on graphs, birth-death chains, convergence to stationarity
2. Constructing Markov chains for sampling: Metropolis–Hastings for hard spheres, connections with statistical inference, coupling and random cluster dynamics
3. Expander Chernoff bound, stochastic domination via coupling Markov chains, applications of stochastic domination and FKG
4. Markov random fields on finite graphs (independent sets and the Ising model), Gibbs measures, phase transitions, correlation decay
5. Rapid mixing: the coupling method and Dobrushin's condition, high-temperature Curie–Weiss, correlation upper bounds for the stationary distribution via mixing
6. Slow mixing: the conductance method, Cheeger's Inequality, Curie–Weiss dynamical phase transition
7. Deducing properties of the stationary distribution using a Markov chain: functional inequalities (e.g. log-Sobolev) for mixing via tensorization, concentration inequalities via Herbst argument; connection between isoperimetry and concentration

Miscellaneous (4 Lectures)

1. Geometry of polynomials: stability theory, negative dependence, random spanning trees
2. Monomer-dimer model via Heilmann–Lieb zero-freeness
3. Fourier analysis over the hypercube, influences
4. Hypercontractivity and sharp thresholds

3 Homework

Each student will have seven free "late days" which can be used on any problem set at any time during the semester, no questions asked. Here are the tentative due dates for the problem sets.

	Out	Due
PSET 1	February 5	February 17 (11:59pm EST)
PSET 2	February 17	February 28 (11:59pm EST)
PSET 3	March 3	March 14 (11:59pm EST)
PSET 4	March 31	April 11 (11:59pm EST)
PSET 5	April 14	April 25 (11:59pm EST)
PSET 6	April 28	May 9 (11:59pm EST)

4 Midterm Exam

There will be a single take-home midterm exam to be completed individually. It will be released tentatively on March 17th, and will be due on March 19th (11:59pm EST). If corrections or adjustments need to be made, announcements will be made on Canvas.

5 Final Project

For the final project, you may work in groups of up to three people. Please submit a write-up PDF, typed in LaTeX, of one of the following:

- **Original Research:** Describe an open problem relevant to the topic of the course and your attempt to solve it. Please give background on motivation, importance, and if applicable, previous works/approaches in that direction. It is OK if your progress does not meet the bar for publication, but please provide complete proofs, references, justification for your approach, etc.
- **Survey:** Please select at least $2-3$ papers to read in depth, and write a summary. The goal here is to have a *unified* perspective of that body of research, as opposed to a list of isolated results. At the most basic level, discuss the underlying context, state the main results, and provide proofs of the key lemmas. However, your report should also contain some of the following elements:
  - Comparisons between different approaches to an open problem. As an example, one ideal version of this is to formulate precisely the key barriers that must be overcome, and why each approach does or does not achieve this. Avoid simply saying "this approach works in X regime of parameters whereas this other approach works in Y regime of parameters".
  - An attempt to apply the techniques to new domains, along with a discussion of why the results are (or aren't) interesting.
  - A preliminary exploration into a connection with a seemingly unrelated field of research.
  - Alternative proofs of key lemmas.
  - Discussion of why certain hypotheses were made, and to what extent they can be relaxed or modified.
  - Simplified exposition. It is OK to weaken the theorem statements, as long as the most novel conceptual contributions are still captured.
  - More "intuitive" exposition, by way of analogies, Ansatz, etc. which are not mentioned in the paper(s). This is particularly valuable if some proofs contain definitions/manipulations/lemmas which really make you wonder "How did they come up with that?"
  - Representative special cases/examples which are not mentioned in the paper(s) and which can be worked through explicitly.
  - Discussion of remaining open problems and why you feel they are important.

Tentative Due Dates

- March 17, 2025: Please submit a half-page initial proposal describing who you plan to work with, and what you plan to do.
- May 9, 2025: Please submit your full report.