6.S891 Algorithmic Counting and Sampling: Probability, Polynomials, and More

(Very Approximate) Syllabus

Fall 2023

This course introduces the modern theory of algorithms for sampling from high-dimensional probability distributions and estimating their partition functions. These fundamental algorithmic primitives lie at the foundation of statistics, physics, and machine learning. We will study a diverse set of algorithmic paradigms for solving these problems based on Markov chains, decay of correlations, geometry of polynomials, and more. We will further study the rich set of analytic, probabilistic, and algebraic tools used to give performance guarantees to these algorithms.

Deliverables

- Three problem sets
- A research-oriented final project

1 Outline of Topics

Below is a very (truly very) approximate list of topics for the course. I'm not sure how much of it we'll get to but we'll see how the course progresses.

Basics of counting, sampling, and Markov chains

- September 7: Overview, models and connections, #P complexity class, Monte Carlo method, counting and sampling reductions
- September 12: Introduction to Markov chains, Glauber dynamics, (path) coupling, colorings
- September 14: Eigenvalues, conductance, functional analytic approach
- September 19: Canonical paths technique, multicommodity flows, ferromagnetic Ising model, high-temperature expansion

The correlation decay method

- September 21: Independent sets/hardcore gas model, phase transitions, spatial mixing, lower bounds for mixing times
- September 26: Tree recursion, correlation decay algorithm, Weitz's result, self-avoiding walk tree

The polynomial/algebraic perspective

- September 28: Multivariate zero-freeness, Asano–Ruelle contractions, Lee–Yang Theorem, connections with phase transitions
- October 3: Barvinok polynomial interpolation, approximating the permanent
- October 5: Matching polynomial, Patel–Regts, Heilmann–Lieb Theorem
- October 10: Cluster expansion, polymer models, ferromagnetic Potts model on expander graphs, #BIS

Returning to Markov chains, spectral independence

- October 12: Spectral independence, local-to-global theorem, connection to high-dimensional expanders
- October 17: Matroids, Trickle-Down Theorem, log-concave polynomials, optimal mixing for matroids
- October 19: Optimal mixing of Glauber dynamics
- October 24: Correlation decay implies spectral independence, hardcore model
- October 26: Zero-freeness implies spectral independence, matchings, even subgraphs
- October 31: Mixing of local Markov chains implies spectral independence, Stein's method for Markov chains, colorings
- November 2: Local coupling technique, even subgraphs, reduction to spatial mixing on trees
- November 7: Sampling Lovasz Local Lemma

Mixing techniques beyond spectral independence

- November 9: Entropic independence, localization schemes
- November 14: Localization schemes continued, field dynamics, Ising models with generic interaction matrices

Variational algorithms

- November 16: Gibbs variational principle, mean-field approximation, convex relaxations
- November 21: Gurvits capacity, permanent, mixed volume
- November 23: No class Thanksgiving holiday
- November 28: Log-concave polynomials revisited, deterministic simply exponential approximation for matroid intersection

End of the course

- November 30: Project presentations
- December 5: Project presentations
- December 7: Project presentations